Error Localization of Complex DFT Codes Using Propagator Method

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Abstract— **In this paper we consider the problem of error localization of complex DFT codes and the reconstruction of message. A propagator-based subspace algorithm is proposed for error localization of quantized DFT codes. The propagator method requires only linear operations and does not require any eigendecomposition of syndrome covariance matrix as in conventional subspace method such as MUSIC etc. The performance of the proposed method is compared with MUSIC algorithm. Simulation results show that the proposed method performs equal to the MUSIC algorithm with less computational complexity.**

Index Terms— **DFT code, propagator, MUSIC, syndrome, frequency estimation, error localization.**

I. INTRODUCTION

DFT (Discrete Fourier Transform) code is a class of real number BCH (Bose-Chaudhuri-Hocqueungem) codes which is used as joint source-channel coding to provide robustness to data loss or corruption in communication channel. DFT code is a (N, K) linear block code whose generator matrix is formed from the inverse DFT (IDFT) matrix. The generator matrix consists of any K columns of IDFT matrix and the remaining N-K columns of IDFT matrix forms the parity check matrix. Since the columns of the IDFT matrix are orthogonal to each other we can find that $\mathbf{G}^H\mathbf{H} = \mathbf{0}$. That is, DFT of every code word becomes zero at fixed set of frequencies called parity frequencies or syndrome frequencies. If the DFT coefficient of the received vector **r** over the syndrome frequencies is non zero (i.e., $\mathbf{r}^H\mathbf{H}\neq\mathbf{0}$) then it indicates that error has occurred.

If the separation between parity frequencies are α, and if α is relatively prime to N, then DFT code is a BCH code in complex field [1]. Through some elementary column operations, if we arrange the generator matrix such that the complex conjugate of every column of the generator matrix also belongs to it, then the BCH code can be made real [1]. A BCH DFT code is a maximum distance separable (MDS) code. The minimum Hamming distance of an (N, K) BCH DFT code is d+1, where d=N-K. So it can correct up to $\lfloor d/2 \rfloor$ sample errors, where $\lfloor x \rfloor$ denotes largest integer no greater than x.

DFT codes are cyclic codes in complex field [1]. There exist two types of DFT codes viz., real DFT (RDFT) codes and complex DFT (CDFT) codes. In [2], and [3] the authors have showed that the error correction in RDFT code is nothing but a complex frequency estimation problem. Frequency estimation techniques, like subspace method and MUSIC (Multiple SIgnal Classification), were applied in [4] for error correction of CDFT codes. In this paper we present a propagator-based subspace algorithm to localize the channel errors, which is more efficient than MUSIC algorithm in terms computations. The simulation results show that the performance of propagator algorithm is as good as MUSIC algorithm.

In this paper we consider only the complex DFT codes, which are formed from the generator matrix $\mathbf{G}=N^{1/2}\mathbf{W}^H_{N\times K}$, where $\mathbf{W}^H_{N\times K}$ is the matrix formed using K columns of IDFT matrix $\mathbf{W}^H_{N\times N}$. And the parity check matrix is given by $\mathbf{H}=\mathbf{W}^H_{N\times N\text{-}K}$ which consists of the remaining N-K columns of $\mathbf{W}^H_{N\times N}$. The code vector is formed as

$$\mathbf{y}=\mathbf{x}\mathbf{G} \qquad (1)$$

Since the message is real, the code word **y** will be conjugate symmetric so it is enough to send only the first half of the code word. At the receiver side the other half will be reconstructed by conjugating the first half. So the channel errors which occur during transmission will also come in conjugate pairs.

II. ERROR LOCALIZATION

Let **r** be the received vector, which is the sum of the transmitted code word and the error vector **e.** The error vector **e** is of dimension 1 × N and has nonzero components at positions where error has occurred. The error **e** consists of two types of noises viz.,

(i) channel noise, which occurs during transmission through the channel and it affects the code word at random locations.
(ii) quantization noise, which occurs due to the quantization of code word and it affects all the locations of the code word

Presence of errors in the received vector is indicated by the nonzero values of syndrome. The syndrome $\mathbf{s}=[s(1),\ s(2),\ldots\ldots,\ s(N\text{-}K)]$ is a row matrix of size 1×N-K and it can be determined by

$$\mathbf{s} = \mathbf{r}^H\mathbf{H} = (\mathbf{y} + \mathbf{e})^H\,\mathbf{H} = \mathbf{e}^H\,\mathbf{H} \qquad (2)$$

The syndrome depends only on the error vector **e,** so it can be used to detect and correct errors in the received vector. Since the channel error has conjugate symmetry, the syndrome **s** will be real and the equation (2) can be written as [11]

$$s(l\text{-}K)=\sum_{j=1}^{v} e_{i_j}\cos(\omega_{i_j} l+\varphi_{i_j})\,,\ l = K, K+1\ldots\ldots, N\text{-}1 \qquad (3)$$

where i_1, i_2, …….. i_v denotes the indices of complex erroneous code word , v denotes the number of complex conjugate pairs which are affected by channel errors, $\omega_{i_k} = (2\pi / N)i_k$, e_{i_k} denotes amplitude of the error and φ_{i_k} denotes phase of the error.

If ω_{i_1} .... ω_{i_v} are known then we can solve the equation (3) to get the error amplitude e_{i_k} and phase φ_{i_k}. Finding ω_{i_1} .... ω_{i_v} is a estimation of real sinusoid frequencies. Thus it is possible to correct the errors once these frequencies are found.

III. FREQUENCY ESTIMATION

Since the message is real, the code word **y** will be conjugate symmetric so we need to send only the first half of the code word. At the receiver side the other half will be reconstructed by conjugating the first half. So the channel errors will also come in conjugate pairs. Therefore the syndrome will be real and the problem of error correction of CDFT codes is analogous to estimating real frequencies buried in quantization noise. In this paper we have used a propagator-based subspace algorithm for estimating real sinusoid frequencies.

A. Formulation of Data Model

In this section the formation of data model is discussed. In subspace based algorithm a vector space is divided in to two orthogonal subspaces. One subspace is spanned by the error locator vector and another subspace is orthogonal complement of it. These two subspaces are obtained from a syndrome covariance matrix. Defining

$$\mathbf{s_1}(k) =[s(k)\ s(k+1)\ s(k+2)\ \ldots.s(k+m\text{-}1)]^T \qquad (4)$$

and

$$\mathbf{s_2}(k) =[s(k)\ s(k\text{-}1)\ s(k\text{-}2)\ \ldots.\ s(k\text{-}m+1)]^T \qquad (5)$$

then a syndrome covariance matrix **R** of dimension $m \times m$, (m>v) is formed as

$$\mathbf{y}(k) = [\mathbf{s_1}(k) + \mathbf{s_2}(k)] \qquad (6)$$

where k = m+1, m+2, ….. , d-m+1.

$$\mathbf{R} = \frac{1}{d-2m+1}\sum_{k=m+1}^{d-m+1}\mathbf{yy}^H \qquad (7)$$

Substituting equations (3), (4) and (5) in (6), we can deduce equation (6) as

$$\mathbf{y}(k) = \mathbf{Az} \qquad (8)$$

where

$$\mathbf{A}= [\ \boldsymbol{a}(\omega_1)\ \ \boldsymbol{a}(\omega_1)\ \ldots.\boldsymbol{a}(\omega_v)],$$

$$\boldsymbol{a}(\omega_i) = [\ 1\ \ \cos(\omega_i)\cos(2\omega_i)\ \ldots..\ \cos((m\text{-}1)\omega_i)\ ]^T,$$

and

$$\mathbf{z}(t) = [e_{i_1}\cos(\omega_1 t),\ e_{i_2}\cos(\omega_1 t),\ldots.,\ e_{i_1}\cos(\omega_v t)]^T$$

B. Propagator Algorithm for Error Localization without quantization Error

Propagator method is a subspace-based method which does not require the eigendecomposition of syndrome covariance matrix, and hence the computation required is much less than the MUSIC algorithm. The matrix **A** is called error locator matrix, and its columns are formed by the error locator vectors $\boldsymbol{a}(\omega_i)$, which span the channel error subspace. Under the hypothesis that **A** is of full rank, v rows of **A** are linearly independent. The other rows can be expressed as a linear combination of these v rows. Thus the error locator matrix **A** of order $m\times v$ can be partitioned as

$$\mathbf{A} = [A_1^H\ \ A_2^H]^H \qquad (9)$$

where $\mathbf{A}_1$ and $\mathbf{A}_2=\mathbf{P}^H\mathbf{A}_1$ are of are matrices of dimensions $v\times v$ and $m\text{-}v\times v$ respectively and where **P** is the propagator matrix of dimension $v\times m\text{-}v$. Using (9), we can partition the syndrome covariance matrix **R** in (7) as

$$\mathbf{R}= [\ \mathbf{R}_1\ \ \mathbf{R}_2] \qquad (10)$$

with $\mathbf{R}_2=\mathbf{R}_1\mathbf{P}$, where $\mathbf{R}_1$ and $\mathbf{R}_2$ are matrices of dimension $m\times v$ and $m\times m\text{-}v$ respectively.

Since we are taking finite number of samples the relation $\mathbf{R}_2= \mathbf{R}_1\mathbf{P}$ does not hold. However, a least-squares solution for the propagator matrix **P** satisfying the relation $\mathbf{R}_2=\mathbf{R}_1\mathbf{P}$ may be obtained by minimizing the cost function

$$J(\mathbf{P}) = ||\mathbf{R}_2\text{-}\mathbf{R}_1\mathbf{P}||^2_F \qquad (11)$$

where $||.||_F$ denotes the Frobenius norm. The cost function J(**P**), being a quadratic (convex) function of **P**, may be minimized to give a unique least-squares solution for **P**:

$$\mathbf{P} = (\mathbf{R}_1^H\mathbf{R}_1)^{-1}\,\mathbf{R}_1^H\mathbf{R}_2 \qquad (12)$$

Let $\mathbf{Q}^H =[\mathbf{P}^H\ \textbf{-I}\ ]$ and **I** is an identity matrix of order $m\text{-}v$, then using equation (9) and the relation $\mathbf{A}_2=\mathbf{P}^H\mathbf{A}_1$ we can write

$$\mathbf{Q}^H\mathbf{A} = 0 \qquad (13)$$

This implies,

$$\mathbf{Q}^H\,\boldsymbol{a}(\omega_i) = 0 \qquad (14)$$

Using equation (14) frequencies ω_i s are found and by solving the consistent system of equations (3) error amplitude and phase are determined.

C. Propagator Algorithm for Error Localization with quantization Error

With quantization error the frequency estimation is same as without quantization error except that the syndrome equation (3) changes as shown

$$s(l\text{-}K)=\sum_{j=1}^{v} e_{i_j}\cos(\omega_{i_j} l+\varphi_{i_j})+\eta_q(l-K) \qquad (15)$$

for $l = K, K+1\ldots\ldots, N\text{-}1$

where n_q is the term which corresponds to the quantization error $\mathbf{e_q}$. The difference between Section B and Section C is that in Section C the error $\mathbf{e} = \mathbf{e_c} + \mathbf{e_q}$, whereas in section B there is only channel error that is $\mathbf{e}=\mathbf{e_c}$. In this case equation (13) and (14) will not hold and estimated frequencies are the values of ω_i's at which equation (13) and (14) reaches v least values.

IV. MESSAGE RECONSTRUCTION

Once we find out error pattern we can subtract it from the received code vector to decode the message at the receiver. Let **r** be the received code word, **y'** be the corrected code word, and **e'** be the estimated error vector then,

$$\mathbf{y'} = \mathbf{r} - \mathbf{e'} \qquad (16)$$

and the decoded message **x'** is found as

$$\mathbf{x'} = (\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}\,\mathbf{y'} \qquad (17)$$

V. SIMULATION RESULTS

In order to check the performance of the proposed method, we have taken (18, 7) DFT code and channel error and quantization error are added to the code word. The channel errors are generated from normal distribution with zero mean and quantization errors are generated from uniform distribution with zero mean. For simulation we took m = 4 and error localization performance for 1, 2 and 3 errors were checked.

The simulation results show the relative frequency of correct localization of all errors versus channel error to quantization error ratio in dB. The relative frequency is the ratio of number of correctly localized error to total number of conjugate error pairs. Figures 1, 2 and 3 show the relative frequency of correct localization for one, two and three errors respectively. It shows that the error localization performance for propagator algorithm is as good as MUSIC algorithm, even though the former requires much less computation than the latter.

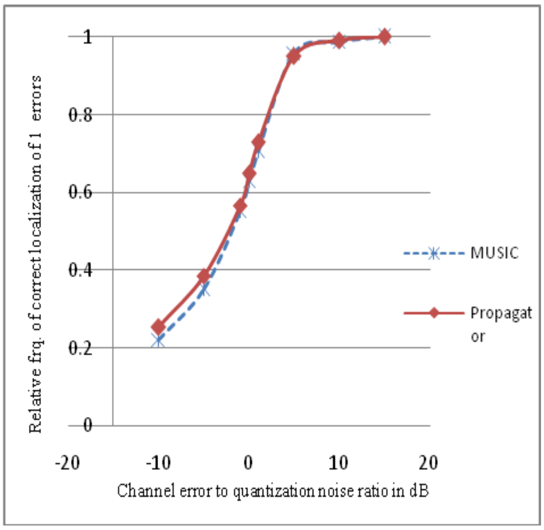

Figure 1. Relative frequency of correct localization for one channel error

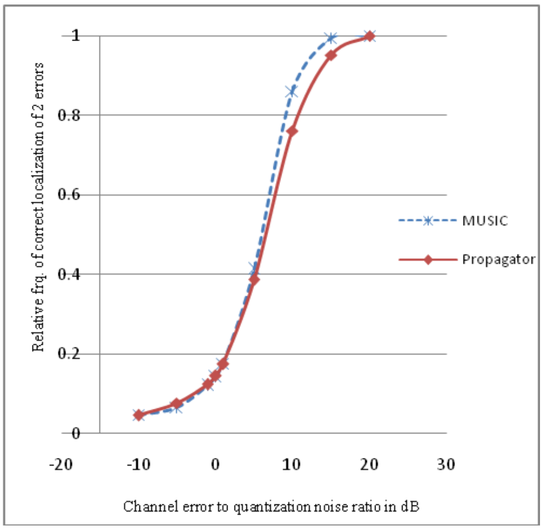

Figure 2. Relative frequency of correct localization for two channel errors

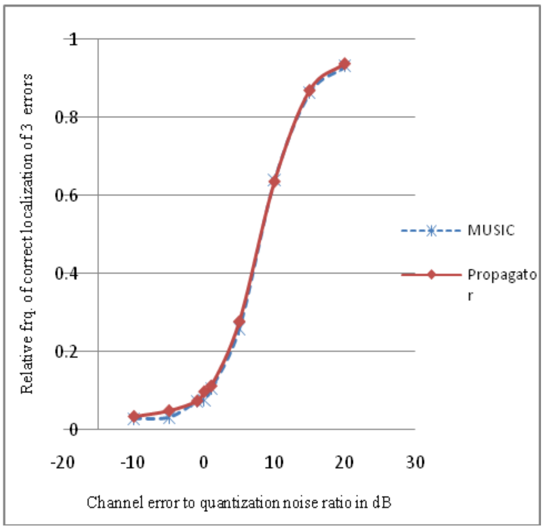

Figure 3. Relative frequency of correct localization for three channel errors

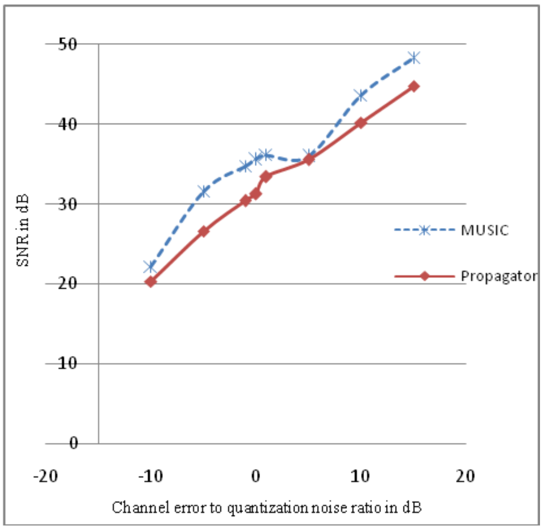

Figure 4. SNR of reconstructed message for one channel error

Figures 4, 5 and 6 show the SNR of the reconstructed signal for different number of errors. The difference between the reconstructed message and the original message is considered as noise. The correct localization of error does not always guarantee lesser noise. As the channel error to quantization error ratio increases SNR also increases. Here also we can see that performance of propagator and MUSIC algorithm are similar.

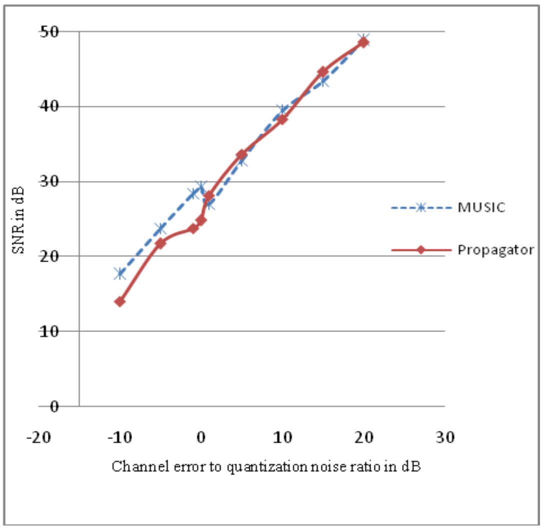

Figure 5. SNR of reconstructed message for two channel errors

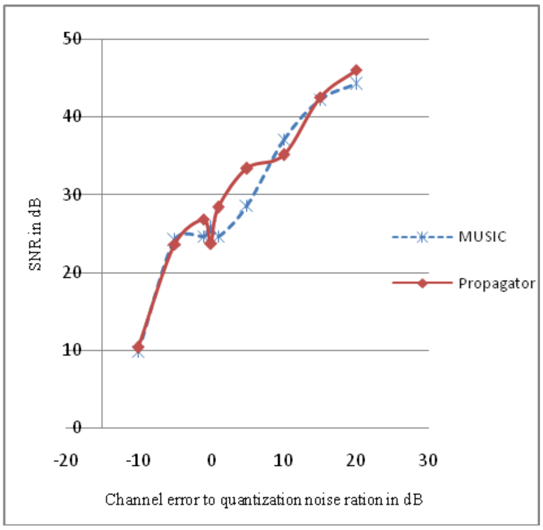

Figure 6. SNR of reconstructed message for three channel errors

VI. CONCLUSIONS

In this paper, the error localization of complex DFT code in a communication network is considered. Error localization of complex DFT is similar to the estimation of real sinusoidal frequencies. A propagator-based subspace algorithm is proposed for estimating the frequencies of real sinusoids. The advantage of the propagator method is that it does not require any eigendecomposition as in MUSIC algorithm. That is the computational complexity of the propagator method is very less compared with MUSIC algorithm. The computer simulation results show that the proposed method is as good as MUSIC with less computation than MUSIC. In this paper we have also shown that the algorithm can be modified to correct quantization errors along with channel errors. Unlike in [11] the proposed method is capable of detecting channel error which occur at 9th position in an (18, 7) code, provided the channel error is real.

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